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Yevick, David; Bardyszewski, Witold; Hoenders, Bernhard

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Comment on "Phonon fluctuation model for flicker noise in elemental semiconductors" [J. Appl. Phys. 52, 2884 (1981)]

David Yevick, Witold Bardyszewski,[a] and Bernhard Hoenders[b]
Department of Theoretical Physics, University of Lund, Sölvegatan 14a, S-223 62 Lund, Sweden

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We note several major inconsistencies in the cited paper.

In a series of papers published in 1981,[1–3] Jindal and van der Ziel attempt to correlate the measured $1/f$ noise in elemental semiconductors with isotope scattering induced fluctuations in the phonon occupation number of long wavelength phonons. As such concept is, at first sight, highly attractive, we felt justified in undertaking a close examination of these papers. Unfortunately, our inspection revealed a series of logical and computational errors, the resolution of which illustrates several important physical points, and which we consequently feel are of value to enumerate here. We will examine Ref. 1, which is the most detailed of Refs. 1–3.

Neglecting minor errors, such as the factor kT in Eq. (1), the first major error of the paper occurs at Eq. (11). Here the author must argue that isotope scattering is the dominant scattering mechanism for very long wavelength phonons, since if some other process becomes important as λ increases, the effective decay time constant in Eq. (4) will eventually be governed by the sum of the inverse time constants of the two processes, and the wavelength dependence of this compound time constant can therefore diverge widely from the assumed λ^4 law. Consider, however, the range of wavelengths and corresponding time constants [Eq. (11)] which, according to the authors, contribute to $1/f$ noise. These time constants are later employed in Eqs. (40) and (48) to determine the range of frequencies for which the $1/f$ law is valid as well as the magnitude of Hooge's constant. Unfortunately, for a phonon of wavelength 8.6 μm, the time constant associated with isotope scattering cited by the authors is 10^6 s, which corresponds to a mean free path of 8×10^6 km. This value is in contradiction with the measured mean free path in germanium of several centimeters at a phonon wavelength of 20 μm.[4,5] Obviously the point is that other processes such as anharmonic effects associated with the crystal surface and interactions with the external heat bath will dominate the scattering of long-wavelength phonons at far smaller wavelengths than 8.6 μm. Further, for a sample with a dimension of less than the mean free path, the smallest allowed momentum value in the crystal is of the order of the inverse sample dimension, which in reality means that for momenta of this order, surface effects become predominant. Therefore, the inverse sample dimension may also act as an effective low momentum cutoff.

The next major error in the paper occurs in Eqs. (18)–(23). Here the objective is to introduce the fluctuations of the phonon number Δn_q from their average value $\bar{n}_q$ into the collision integral of the Boltzmann equation for electrons. While Eq. (18) is correct up to an interchange of k and k' and the addition of several parentheses, the subsequent discussion, which allows the identification of a relaxation time for the electron momentum, is unjustifiable. The problem here is that we are interested in the additional contribution to the nonequilibrium electron density function induced by temporal variations in the phonon number. Consequently, it is not valid to eliminate the terms in Eq. (18) explicitly proportional to Δn_q as being small compared to the last term in the equation, which is proportional to $\bar{n}_q + \Delta n_q$ since the part of this last term proportional to n_q must first be subtracted before effecting the comparison. In other words, in Eq. (19) (which should contain an absolute value on the left-hand side) n_q must be replaced by Δn_q on the right-hand side. It is therfore perhaps not surprising that the introduction of the "attenuation constant," α_a, in Eq. (23) seems to be in contradiction to the laws of statistical physics, expressed in Eq. (2).

Although the above considerations would appear to invalidate the subsequent results of the paper, let us for the sake of discussion accept the author's contention that the effects of phonon fluctuations can be incorporated simply by replacing the average phonon number $\bar{n}_q$ by the fluctuating number n_q in the equations for the momentum relaxation time or electron mean free path. It is then still doubtful, because of the relatively large statistical fluctuations of the phonon number, that the ensemble average $\langle \delta l \delta l / l \cdot l \rangle$ of Eq. (35), where $l(\theta,\phi)$ is the angular-dependent mean free path, can be replaced by the quotient of the individual ensemble averages of the numerator and denominator. More importantly, however, let us now contrast Eqs. (37)–(50) with the proper procedure for obtaining the current-current correlation function. First, the current should be expressed

[a] Permanent address: Institute of Theoretical Physics, University of Warsaw, Hoza 69, 00 681 Warsaw, Poland.

[b] Permanent address: Department of Applied Physics, University of Groningen, Universiteitscomplex Paddepoel, Nijenborgh 18, 9747 AG Groningen, The Netherlands.

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as an integral over momentum of the mean free path weighted by the component of the electron velocity along the direction of the electric field times the energy derivative of the Fermi factor.[6] The ensemble average $\langle J(t)J(0)\rangle$ is then given as a six-dimensional integral over the electron momenta $\bar{k}_1$ and $\bar{k}_2$ involving $\langle \delta l(t)\delta l(0)\rangle$, which can in turn be related (according to the incorrect arguments cited in the previous paragraph) to $\langle n_{q_1}(t)n_{q_2}(0)\rangle$ by means of an integral over solid angles $d\Omega_1'$ and $d\Omega_2'$. Here the primed variables refer to the direction of the electrons in the final state with respect to the initial state momenta after scattering by phonons with momenta $\bar{q}_1$ and $\bar{q}_2$, respectively. The final expression, which appears extremely difficult to evaluate analytically, may be incorrectly simplified by assuming that only mean free paths of electrons with the same vector momenta which are scattered through the same angle are correlated; that is, that

$$\langle \delta l(\bar{k}_1-\bar{k}_1'),\delta l(\bar{k}_2-\bar{k}_2')\rangle \sim \delta(\bar{k}_1-\bar{k}_2)\cdot\delta(\bar{k}_1'-\bar{k}_2')$$

rather than $\delta(\bar{k}_1-\bar{k}_1'-\bar{k}_2+\bar{k}_2')$, in accordance with the correlation law for phonon number fluctuations of Eqs. (4) and (5) [in the preceding formulas we have represented $l(\theta,\phi)$ of the references in the more correct form $l(\bar{q})$]. This assumption is introduced implicitly in Fig. 1 of Ref. 1 and in Eqs. (35) and (36), where $S_{l(\theta,\phi)}$ is treated as a function of only the angular variables corresponding to one electron momentum, $\bar{k}_1=\bar{k}_2=\bar{k}$, rather than the two different momenta $\bar{k}_1$ and $\bar{k}_2$. As a result of this unjustified assumption, Jindal and van der Ziel are able to perform first the ensemble average, then the average over initial electron momenta in a given direction, and finally the average over final electron momenta. Further, the initial momenta averaging is performed with a weighting factor proportional to the number of phonons in a given state $\bar{q}$ rather than to the energy derivative of the Fermi distribution function for electrons, as mandated by the Boltzmann equation. It is instructive to note in this context that the units of the right- and left-hand sides of Eq. (49) do not agree. Regarding the average over final states, we observe that even if we accept the stated form of $S_{l(\theta,\phi)}$, the ensemble averaging procedures and mathematical manipulations involved in Eqs. (41) and (42) appear incorrect.

In conclusion, we have identified several defects in the theoretical treatment of phonon scattering processes given by Jindal and van der Ziel. These are (1) the assumed range of the λ^4 law for the decay times of long-wavelength phonons, (2) the assumption of small functuations for the phonon number associated with a given phonon momentum relative to the average phonon number in the mode, (3) the disregard of seemingly important terms in the Boltzmann equation for the fluctuating electron distribution function, and (4) the incorrect averaging procedure over the initial and final states of electrons as well as the incorrect ensemble averaging procedure.

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Reply to "Comment on 'Phonon fluctuation model for flicker noise in elemental semiconductors'" [J. Appl. Phys. 52, 2884 (1981)]

R. P. Jindal
AT&T Bell Laboratories, 600 Mountain Avenue, Murray Hill, New Jersey 07974

A. van der Ziel
Electrical Engineering Department, University of Minnesota, Minneapolis, Minnesota 55455

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Phonon fluctuation model for flicker noise in elemental semiconductors is consistent with then existing data. New experimental results may necessitate a reinterpretation of some of the axioms of the model. A further improvement in the model can result by reviewing some of the assumptions involved.

After a period of nearly five years we are pleased to see an interest developing again in the phonon fluctuation model[1-3] for explaining $1/f$ noise in elemental semiconductors. In 1980 there was a lot of confusion about mobility fluctuation $1/f$ noise. In view of this confusion, coupled with new experimental results, it is not surprising that our papers are somewhat dated. Nevertheless, these papers served a useful purpose.

Yevick, Bardyszewski, and Hoenders[4] deserve our appreciation for taking the time to go through the above derivations themselves and highlighting the drawbacks of the theory. The purpose of this communication is to examine these points in retrospect, clearly differentiating between proofs and assumptions, and in relation to the present $1/f$ noise theories.

In Ref. 1 we state that $1/f$ noise can be interpreted as

